Spring 2004

Physics 137B Section 1: Problem Set #3
Due: 5PM Friday Feb 12 in the appropriate dropbox inside 251 LeConte (the "reading room")

Suggested Reading for this Week:

- Bransden and Joachain (B& J) sections 8.1-8.2
- B& J section 12.1

Homework Problems:

1. B& J problem 8.6 (Note: This is one of the few cases where using the explict forms for the harmonic oscillator wave function is better than using the operators a and $a^\dagger$)

2. B& J problem 8.7

3. Consider a particle confined in a two-dimensional infinite square well with faces at $x = 0$: $x = L$ and $y = 0 : y = L$. The doubly degenerate eigenstates appear as

$$\psi_{np}(x, y) = \frac{2}{L} \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{p\pi y}{L}\right)$$

with energy $E_{np} = E_1(n^2 + p^2)$. How do these energies change under the perturbation

$$H' = 10^{-3} E_1 \sin\left(\frac{\pi x}{L}\right)$$

4. The Hamiltonian for a quantum mechanical dumbbell is

$$H = \frac{L^2}{2I}$$

where I is the moment of inertial of the dumbbell. The eigenstates of this system are thus

$$E_\ell = \frac{\hbar^2 \ell(\ell + 1)}{2I}$$

and for a given ℓ is $(2\ell + 1)$-fold degenerate. (See B& J pages 290-292 if you are not familiar with this problem.) In the event that the dumbbell is equally and oppositely charged at its ends, it becomes a dipole. The interaction energy between such a dipole and a constant, uniform electric field $\vec{E}$ is

$$H' = -\vec{d} \cdot \vec{E}$$

where $\vec{d}$ is the dipole moment of the dumbbell. Show that to terms of first order, this perturbing potential *does not separate* the degenerate E_ℓ eigenstates.

5. In class we solved the Stark Effect problem, a hydrogen atom with the perturbing term in the Hamiltonian

$$H' = e\mathcal{E}z$$

and saw that for the fourfold degeneracy of the $n = 2$ states is partially lifted (See Figure 12.1 in B& J). Show explicitly that the 4×4 matrix of H' is diagonal in the basis

$$\begin{aligned}
\xi_1 &= \psi_{211} \\
\xi_1 &= \frac{1}{\sqrt{2}}(\psi_{200} - \psi_{210}) \\
\xi_1 &= \frac{1}{\sqrt{2}}(\psi_{200} + \psi_{210}) \\
\xi_1 &= \psi_{21-1}
\end{aligned}$$

where $\psi_{n\ell m}$ is the wave function of the unperturbed hydrogen atom with eigenvalues n, ℓ, m.